Extended CreditGrades Model with Stochastic Volatility and Jumps

Artur Sepp

Mail: artursepp@hotmail.com, Web: www.math.ut.ee/~spartak

Department of Industrial Engineering and Management Sciences
Northwestern University
2145 Sheridan Road, Evanston, IL 60208-3119

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Abstract

We present two robust extensions of the CreditGrades model: the first one assumes that the variance of returns on the firm’s assets is stochastic, and the second one assumes that the firm’s asset value process follows a double-exponential jump-diffusion. We derive closed-form formulas for pricing equity options on a reference firm in this setting and for calculating the survival probability of this firm during a finite time horizon. We apply these models for modeling credit default swap (CDS) and equity default swap (EDS) spreads. We calibrate our models to General Motors options data and discuss the results. It follows that both models provide a good fit to the data and lead to non-zero short-term CDS spreads.

The contribution of this paper is threefold. First, we incorporate jumps into the CreditGrades model. Although the Merton’s model with jump risk has already been considered in a number of studies, there was no reference on how to connect the default risk with equity risk, i.e. how to estimate default probabilities using equity options. Secondly, we consider the stochastic variance of the firm’s value in the CreditGrades model and make a connection to equity options. This model seems to be new. Finally, we consider incorporating random default barriers and provide an alternative to the CreditGrades approach on how to deal with random default barriers by computing survival probabilities and option prices. This approach is based on the convexity adjustment and it can applied to diffusions with stochastic variance and jumps.

**Keywords:** Credit risk, CreditGrades model, jump diffusion processes, stochastic volatility, credit default swap spreads, equity default swap

1 Introduction

Credit risk is the risk that an obliger fails to honor its obligations. Coupled with a recent wave of bankruptcies, credit derivatives market has been sparking and raising demand for more sophisticated methods to evaluate and manage the credit risk.
One of the popular approaches to manage credit risk is Merton’s (1974) model. In this model, the company defaults if the value of its assets becomes less than its promised debt repayment at maturity time $T$. Among others, Black-Cox (1976) and Leland-Toft (1996) extended Merton’s model to account for the possibility that default may happen prior to maturity date $T$. Other extensions propose stochastic interest rates (Longstaff-Schwartz (1995)), stochastic default barriers (Finger et al (2002)), jumps in the firm’s value dynamics (Zhou (1997, 2001), Hilberink-Rogers (2002), Lipton (2002)). For a comprehensive review of the Merton model and its ramifications we refer to the recent books by Bielecki-Rutkowski (2002), Schönbucher (2003), and Lando (2004).

One of the drawbacks of the Merton’s model is that it provides no connection to equity market, in particular, to equity options. This drawback was circumvented by CreditGrades (equity-to-credit) model, which became quite popular in the credit derivatives market. The CreditGrades model was jointly developed by CreditMetrics, JP Morgan, Goldman Sachs, and Deutsche Bank, and it has subsequently been copywritten. A detailed description of the CreditGrades model is presented in Stamicar et al (2005), Finger et al (2002), Finkelstein (2001). In short, this approach supplements the Merton’s model (1974) by providing a link to equity market and, in particular, to equity options. CreditGrades model is based on the assumption that the firms value follows a pure diffusion with a stochastic default barrier which is introduced to make the model consistent with high short-term CDS spreads.

Another essential problem in the Merton’s model is the so-called predictability of default, which is discussed in details in Lando (2004) and Elizalde (2005). In a nutshell, since most of the structural models assume a continuous diffusion processes for the firms value dynamics and complete information about the firm’s value and default barrier, the actual distance from the current value of the firm to the default barrier implies the “nearness of default”. Accordingly, if the current market value of the firm is far away from its default barrier, the probability of default in the short-term is close to zero, because the firm’s value process needs time to reach the default point. The knowledge of that distance to default and the fact that the firm’s value follows a continuous diffusion process makes the firm’s default a predictable event, i.e. default does not come as a surprise. Since the probability of default for a continuous diffusion process in a very short time interval is close to zero, this leads to very low short-term credit spreads, which are close to zero. In contrast, it is observed in the markets that even short-term credit spreads are non-zero, incorporating the possibility of an unexpected default or deterioration in the firms credit quality.

One of the approaches to deal with the predictability of the default is to include jumps in the firm’s value process. Zhou (1997, 2001), Hilberink-Rogers (2002), Lipton (2002) deal with structural models in which the firms dynamics incorporates a jump risk. However, in general this approach is not analytically tractable, since the first-exit time densities for jump-diffusions are rather difficult to obtain in closed-form. In addition, it was not discussed how to connect these models to equity markets.

Recently, Sepp-Skachkov (2003) and Sepp (2004) developed a tractable approach
for pricing single and double barrier options under a double-exponential jump-diffusion. Here, we extend their framework for analytical evaluation of equity options in the CreditGrades model with double-exponential jumps and stochastic volatility. Also, we derive formulas for the default probability and consider pricing credit default swaps (CDS) and equity default swaps (EDS).

This paper is organized as follows. In Section 2, we introduce the notations, shortly present the CreditGrades model, and discuss the proposed models for the firm’s value dynamics. In Section 3, we consider the pricing of equity options and derive closed-form formulas for the value of a call option under proposed dynamics. In Section 4, we develop analytical solutions for the firm’s survival probability and default-time densities. In Section 5, we treat incorporating random barriers to the default problem. In Section 6, we consider pricing CDS and EDS in this framework. In Section 7, we apply our models to the implied volatility surface of General Motors options and illuminate the results.

2 Formulation

Here we fix notations and briefly present the CreditGrades model. Detailed description of this model can be found in Stamicar et al (2005), Finger et al (2002), Finkelstein (2001).

We assume a finite time horizon $T$ and suppose a probability space $(\Omega, \mathcal{F}, \mathbb{P}, \mathbb{Q})$ supporting standard Brownian motions $Z, Z''$ and a Poisson process $N$, where $\mathbb{Q}$ is a risk neutral probability measure and $\mathbb{F} = (\mathcal{F}_s)_{0 \leq s \leq t}$ is the information set generated by $Z, Z''$, and $N$ up to time $t$, $0 \leq t \leq T$.

We consider a certain reference firm and use the following notation: $S(t)$ denotes the firm’s equity price per share, $V(t)$ denotes the firm’s asset value per share, $B(t)$ denotes the firm’s total debt per share, and $R$ ($0 < R \leq 1$) denotes the recovery rate of firm’s debt.

We assume that the firm’s debt $B(t)$ is deterministic:

$$B(t) = B(0)e^{\int_0^t (r(s) - d(s))ds},$$

(2.1)

where $r(t)$ is a deterministic risk-free interest rate, and $d(t)$ is a deterministic dividend yield on the firm’s assets.

We assume that the asset value of the firm, denoted by $V(t)$, is driven by a suitable stochastic process under the risk-neutral measure $\mathbb{Q}$ and the firm defaults when its value hits the deterministic barrier $D(t)$ defined by $D(t) = RB(t)$. Accordingly, the time of default on time interval $(t_0, T]$ is defined as

$$\iota = \inf\{\tau \in (t_0, T] : V(\tau) \leq D(\tau)\},$$

(2.2)

with $\iota$ being an $\mathbb{F}$-stopping time.

We define the firm’s equity per share $S(t)$ by:

$$\begin{cases} 
S(t) = V(t) - D(t), & \text{if } \iota > t \text{ and} \\
S(t) = 0, & \text{otherwise.}
\end{cases}$$

(2.3)
From representation (2.3) we can induce an equivalent time of default:
\[ \varsigma = \inf\{\tau \in (t_0, T] : S(\tau) \leq 0\}. \tag{2.4} \]

The specification (2.3) provides a possible way to estimate parameters of the asset’s value process \( V \), which is not directly observable, by linking it to equity, for which market data are usually available.

Let us also note that, in the CreditGrades model, the asset’s value process \( V(t) \) cannot be interpreted as the market value of the firm, since by the financial standpoint the market value of the firm is the sum of the market value of its equity and debt. Accordingly, in specification (2.3), \( V(t) \) is rather interpreted as an underlying state variable related to the asset value process. The default barrier \( D(t) \) can be interpreted as the recovery part of the debt. Since the initial value of \( D(t) \) can be estimated from the balance sheet or market data, the initial value of \( V(t) \) is calculated as \( V(0) = S(0) + D(0) \).

Furthermore, the CreditGrades model assumes that the default barrier \( D(t) \) is a random variable with log-normal distribution and suitably chosen parameters. This assumption leads to the unpredictability of the default event and thus to higher shorter term credit spreads. At this time we consider a deterministic barrier \( D(t) \) and we will treat random barriers in Section 7, where we will employ a convexity adjustment formula to deal with random barriers.

2.1 Firm’s Value Dynamics

In our paper we consider three possible models for the firm’s value dynamics: 1) geometric Brownian motion (regular diffusion), 2) geometric Brownian motion with stochastic variance uncorrelated with the firm’s value dynamic, 3) double-exponential jump-diffusion. In this section, we discuss these models in detail.

Our core assumption is that the firm defaults when the stock price hits zero, which means that zero is an absorbing barrier for price process \( S(t) \), i.e. as soon as \( S(t) \) reaches zero it is stuck there forever. This implies that European call and put options must be priced as the corresponding down-and-out barrier call and put options with the down barrier set at \( S(t) = 0 \). We note that the assumption of the absorbing barrier is more complicated to deal with than the alternative assumption that default can only happen at the maturity time \( T \), i.e. the firms default only if \( S(T) \leq 0 \). The former assumption is evidently more realistic but is requires a lot of analytical efforts since the absorbing transition densities are available only for a limited (but nevertheless very interesting) class of stochastic processes.

2.1.1 Regular Diffusion

The basic version of the CreditGrades model assumes that the firm’s follows a geometric Brownian motion with time-dependent parameters:
\[ dV(t)/V(t) = (r(t) - d(t))dt + \sqrt{\nu(t)}dZ(t), \quad V(0) = S(0) + D(0), \tag{2.5} \]
where $\nu(t)$ is variance of returns on the firm’s assets. Under the above assumptions, prior to the default the stock price follows the so-called shifted log-normal diffusion:

$$dS(t) = (r(t) - d(t))S(t)dt + \sqrt{\nu(t)}(S(t) + D(t))dZ(t), \quad S(0) \text{ given.} \tag{2.6}$$

Introducing a new process for $x = \ln(S(t) + D(t))$ and using Ito’s lemma, we can show that the solution to the stock price dynamics prior to the default is given by:

$$S(t) = (S(0) + D(0))e^{\int_0^t (r(s) - d(s) - \frac{1}{2}\nu(s))ds + \int_0^t \sqrt{\nu(s)}dZ(s)} - D(t). \tag{2.7}$$

Thus, the distribution of stock price under the risk-neutral measure is shifted log-normal (equivalently, the distribution of $S(t) + D(t)$ is log-normal) with the transition probability density function given by

$$p(S(t), t; S'(T), T) = \frac{1}{\sqrt{2\pi \hat{\tau}(S'(T) + D(T))}} \times$$

$$\exp \left\{ - \frac{\left( \ln \left( \frac{S(t) + D(t)}{S'(T) + D(T)} \right) + \int_t^T (r(s) - d(s))ds - \frac{1}{2} \hat{\tau} \right)^2}{2\hat{\tau}} \right\}, \tag{2.8}$$

with $\hat{\tau} = \int_t^T \nu(s)ds$.

The shifted distribution implies negative values of the stock price with positive probability. Using PDF (2.8), we can check that $\mathbb{E}[S(T)] = S(t)e^{\int_t^T (r(s) - d(s))ds}$ ensuring that the discounted price process is a martingale under the risk-neutral measure.

From (2.7), we see that the default does not occur up to time $T$ as long as for all $t \in (0, T]$

$$(S(0) + D(0))e^{\int_0^t (r(s) - d(s) - \frac{1}{2}\nu(s))ds + \int_0^t \sqrt{\nu(s)}dZ(s)} > D(0)e^{\int_0^t (r(s) - d(s))ds}, \tag{2.9}$$

which is equivalent to:

$$(S(0) + D(0))e^{-\frac{1}{2} \int_0^t \nu(s)ds + \int_0^t \sqrt{\nu(s)}dZ(s)} > D(0). \tag{2.10}$$

Accordingly, in this specification the default event does not depend on the risk-neutral drift. Another important conclusion is that default probabilities only depend on the ratio between $V(0) = S(0) + D(0)$ and $D(0)$, and not on the firm’s value and its debt separately.

In Figure 2.1, we illustrate the relationship between the probability density of $S(T)$ and the leverage coefficient $L = V(0)/D(0)$. Clearly, there is an inverse relationship between the level of leverage and the default probabilities - higher values of the leverage coefficient imply lower default probabilities and vice versa.

As a direct implication of the dynamics (2.6), it follows that for a given firm’s asset volatility, $\nu(t)$, the local stock volatility, $\nu_S(t)$, depends on the local stock price:

$$\sqrt{\nu_S(t)} = \sqrt{\nu(t)}\frac{S(t) + D(t)}{S(t)}. \tag{2.11}$$
The lower is the leverage, the greater is the likelihood that $S'(T)$ attains negative values. Thus, the CreditGrades model naturally introduces the volatility skew. However, it is unable to produce the volatility smile, for example, in case when the local volatility is symmetric around the at-the-money volatility. Accordingly, to deal with the smile one needs to consider either stochastic variance or jumps.

### 2.1.2 Diffusion with Stochastic Variance

In this model, which is based on the Heston’s (1993) approach to stochastic volatility, we assume that the firm’s value is driven by a diffusion with stochastic variance that is uncorrelated with the firm’s value process:

$$
\begin{align*}
\frac{dV(t)}{V(t)} &= (r(t) - d(t))dt + \sqrt{\nu(t)}dZ(t), \quad V(0) = S(0) + D(0) \\
\frac{d\nu(t)}{\nu(t)} &= \kappa_{\nu}(\nu_{\infty} - \nu(t))dt + \sqrt{\nu(t)}dZ^\nu(t), \quad \nu(0) = \nu_0,
\end{align*}
$$

(2.12)

where $\nu(t)$ is now a stochastic variance on the firm’s returns, $\nu_{\infty}$ is a long-term variance, $\kappa_{\nu}$ is a mean-reverting rate, $\varepsilon_{\nu}$ is a volatility of instantaneous variance, $Z(t)$ and $Z^\nu(t)$ are independent Wiener processes.

Now, the stock price follows the shifted lognormal diffusion with stochastic variance:

$$
\begin{align*}
\frac{dS(t)}{S(t)} &= (r(t) - d(t))S(t)dt + \sqrt{\nu(t)}(S(t) + D(t))dZ(t), \quad S(0) \text{ given,} \\
\frac{d\nu(t)}{\nu(t)} &= \kappa_{\nu}(\nu_{\infty} - \nu(t))dt + \sqrt{\nu(t)}dZ^\nu(t), \quad \nu(0) = \nu_0,
\end{align*}
$$

(2.13)
The motivation behind this model is that we cannot estimate the variance of the firm’s asset returns for sure, thus we can allow some unpredictability for the variance of the firm’s returns and deal with it appropriately. Another important consequence of this model is that, although we assume no correlation between the asset value and variance for the sake of analytical tractability, the leverage effect (which means that lower values of firm’s assets result in an increased level of variance, while higher values result in a decreased level of variance) is introduced in the above model since the local variance of the firm’s dynamics does depend on the leverage ratio, \((S(t) + D(t))/S(t)\).

Let us recall that the traditional Heston model with zero correlation implies a symmetric smile (Lewis (2000)). However, in our process (2.13) the smile is asymmetric due to the the leverage ratio. The case of asymmetric smile is in agreement with empirical observations for equity options.

One of the possible drawbacks of this model is that not all parameters are directly observable on the markets, so we need to back out the model parameters from market prices of available securities. Fortunately, the availability of closed-form solutions for the values of call and put options as well as for the survival probability makes the calibration procedure less involved.

### 2.1.3 Double-Exponential Jump-Diffusion

Finally, we consider the firm’s dynamics with double-exponential jumps:

\[
\begin{align*}
\frac{dV(t)}{V(t)} &= (r(t) - d(t) - \lambda \nu(t))dt + \sqrt{\nu(t)}dZ(t) + (e^J - 1)dN(t), \\
V(0) &= S(0) + D(0),
\end{align*}
\]

(2.14)

where \(N(t)\) is a Poisson process with deterministic intensity \(\lambda \nu(t)\), and \(J\) is a random jumps with double-exponential distribution PDF \(\varpi(J)\):

\[
\varpi(J) = \varpi^+(J) + \varpi^-(J) = q^+ \frac{1}{\eta^+} e^{-\frac{J}{\eta^+}} 1_{\{J\geq 0\}} + q^- \frac{1}{\eta^-} e^{\frac{J}{\eta^-}} 1_{\{J< 0\}},
\]

(2.15)

where \(1 > \eta^+ > 0, 1 > \eta^- > 0\) are mean sizes of positive and negative jumps, respectively; \(q^+\) and \(q^-\) represent the probabilities of positive and negative jumps: \(q^+, q^- \geq 0, q^+ + q^- = 1\); and \(\alpha\) is chosen to make the discounted firm value process a martingale:

\[
\alpha = \mathbb{E}[e^J - 1] = \frac{q^+}{1 - \eta^+} + \frac{q^-}{1 + \eta^-} - 1.
\]

(2.16)

The double-exponential jump-diffusion was introduced by Kou (2002) and it has subsequently been studied by Kou-Wang (2003), Lipton (2002), Sepp-Sckachkov (2003), Sepp (2004), who employed nice properties of this jump-diffusion, such as the availability of an analytical solution for the first-passage time density in Laplace domain, to obtain closed-form formulas for barrier and lookback options. Here, we extend their results to the CreditGrades model.
In this model, the stock price follows the shifted log-normal diffusion with double-exponential jumps:

\[
\frac{dS(t)}{S(t)} = \left( (r(t) - d(t))S(t) - \lambda \alpha \nu(t) (S(t) + D(t)) \right) dt + \sqrt{\nu(t)} (S(t) + B(t)) dZ(t) + (S(t) + D(t)) (e^J - 1) dN(t), \quad S(0) \text{ given.} \tag{2.17}
\]

The solution to the above SDE is:

\[
S(t) = (S(0) + D(0)) e^{\int_0^t (r(s) - d(s)) ds + \int_0^t \sqrt{\nu(s)} dZ(s) + \sum_{n=1}^N J_n - D(t)}, \tag{2.18}
\]

where \(J_n\) are i.i.d. random variables with PDF \(\varpi(J)\).

By construction, the intensity of jumps is dependent on the variance and, as a result, on the local leverage ratio \(\frac{S(t) + D(t)}{S(t)}\). This is a realistic assumption, since for lower asset prices returns are more volatile and thus they are subject to greater jump risk.

The motivation behind this model is that it introduces the unpredictability of the default event, i.e. default can happen short after the contract inception. This leads to high credit spreads for even very short maturities. Although not all model parameters are directly observable, the availability of closed-form solutions for equity options and survival probabilities makes the model calibration feasible.

### 3 Equity Options

In this section, we will study the pricing of equity options under the above-described models. For brevity, we concentrate on call options. Put options can be handled via the same technique or they can be priced by the put-call parity.

Under risk-neutral valuation, the value of a call option, \(W(t, S)\), can be represented by:

\[
W(t, S) = \mathbb{E}^Q \left[ e^{-\int_t^T r(s) ds} \max\{S(T) - K, 0\} \mathbbm{1}_{\{S(T) > K\}} \mid \mathcal{F}_t \right]. \tag{3.1}
\]

Accordingly, the value of a call \(W(t, V)\) is equivalent to a down-and-out barrier call with strike \(K\) and lower barrier set at \(S = 0\).

#### 3.1 Regular Diffusion

For the dynamics (2.6), \(W(t, S)\) solves the following PDE:

\[
\begin{cases}
W_t + \frac{1}{2} \nu(t)(S + D(t))^2 W_{SS} + (r(t) - d(t)) SW_S - r(t)W = 0, \\
W(t, 0) = 0, \quad W(T, S) = \max\{S - K, 0\}.
\end{cases} \tag{3.2}
\]

We solve this PDE by an application of the method of images. For brevity, we provide the details of our solution method in Appendix A1 and here we state the
final result for the value of a call option under the regular diffusion:

\[ W(t, S) = C_{BS}(t, T, S(t) + D(t), K + D(T)) \]

\[ = \frac{S(t) + D(t)}{D(t)} C_{BS}(t, T, \frac{(D(t))^2}{S(t) + D(t)}, K + D(T)). \]  

(3.3)

where \( C_{BS}(t, T, S, K) \) is the Black-Scholes price of a call option with maturity \( T \), strike \( K \) on the underlying with price \( S \) and average variance, interest rate and dividend yield calculated by: \( \bar{v} = \frac{\int_0^T \nu(s)ds}{T-t} \), \( \bar{r} = \frac{\int_0^T r(s)ds}{T-t} \), and \( \bar{d} = \frac{\int_0^T d(s)ds}{T-t} \), respectively.

Formula (3.3) resembles a standard formula for down-and-out barrier call (note there is no terms with \( \bar{r} \) and \( \bar{d} \) in the power of \( \frac{S(t) + D(t)}{D(t)} \) since by construction the default barrier does not depend on the risk-neutral drift). As a result, the CreditGrades model provides a simple and robust solution to the value of equity options.

### 3.2 Diffusion with Stochastic Variance

Now we consider the stock price dynamics with stochastic variance (2.13). For this model, the value function of a call option,  

\[ W(t) = V(t, V), \]  

solves the following equation:

\[
\begin{cases}
W_t + \frac{1}{2} \nu(t)(S + D(t))^2 W_{SS} + (r(t) - d(t))SW_S + \kappa v(S) - \nu W_{\nu} + \frac{1}{2} \xi v W_{\nu
\nu} - r(t)W = 0, \\
W(t, 0) = 0, W(T, S) = \max\{S - K, 0\}.
\end{cases}
\]  

(3.4)

We solve this PDE by a combination of the Fourier transform and the method of images. A similar technique was employed by Lipton (2001) for pricing double-barrier options in the Heston’s model with zero correlation. We present the derivation of the solution to this equation in Appendix A2. Here we state the final result:  

\[ W(t, S) = (D(T) + K)e^{-\int_t^T r(s)ds}Z(\tau, y), \]  

where \( y = \ln \left( \frac{S + D(t)}{D(T) + K} \right) + \int_t^T (r(s) - d(s))ds \), \( b = \ln \left( \frac{D(t)}{D(T) + K} \right) + \int_t^T (r(s) - d(s))ds \), and  

\[ Z(\tau, y) = e^y - e^b + \frac{e^{\frac{1}{2} y}}{\pi} \int_0^\infty \frac{e^{A(t,k) + B(t,k)v}(\cos(yk) - \cos((y - 2b)k))}{k^2 + \frac{1}{4}} dk, \]  

(3.5)

with  

\[ B(t, k) = -(k^2 + \frac{1}{4}) \frac{1 - e^{-\tau(T-t)}}{\psi_+ + \psi_+ e^{-\tau(T-t)}}, \]

\[ A(t, k) = -\frac{\kappa v}{\xi v} \left[ (T-t)\psi_+ + 2 \ln \left( \frac{\psi_- + \psi_+ e^{-(T-t)\xi}}{2\xi} \right) \right], \]  

(3.7)

\[ \psi_\pm = \mp \kappa v + \xi \xi, \; \xi = \sqrt{\kappa v^2 + \xi v^2 (k^2 + \frac{1}{4})}. \]

Integral in (3.6) is pretty easy to evaluate numerically (for large \( k \) it is an exponentially decaying function of \( k \)). Thus, we obtain a closed-form solution for call option values in the extended CreditGrades model with stochastic variance.
3.3 Double-Exponential Jump-Diffusion

Finally we consider the stock price process driven by the double-exponential jump-diffusion (2.17). Under this model the value of a call option, \( W(t,S) \), solves the following PIDE:

\[
\begin{align*}
W_t + \frac{1}{2} \nu(t)(S + D(t))^2 W_{SS} + (r(t) - d(t)) SW_S + \\
+ \lambda \nu(t) I^{-\infty}[W((S + D(t))e^t) - W(S + D(t))] \varphi(J)dJ - r(t)W &= 0, \\
W(t,0) &= 0, W(T, S) = \max\{S - K, 0\}.
\end{align*}
\]

We solve this equation as follows. First, we introduce \( x = \ln\left(\frac{S + D(t)}{D(t)}\right) \), \( a = \ln \frac{D(T) + K}{D(t)} \), \( W(t, V) \rightarrow G(t, x) = e^{\gamma T} r(s) ds W(t, V) / D(T) \):

\[
\begin{align*}
G_t + \frac{1}{2} \nu(t) G_{xx} + \nu(t) G_x + \lambda \int_{-\infty}^{\infty} [G(x + J) - G] \varphi(J) dJ &= 0, \\
G(t, 0) &= 0, G(T, x) = \max\{e^x - e^a, 0\}.
\end{align*}
\]

where \( \mu = -\frac{1}{2} - \lambda \alpha \).

Next, we introduce \( x \rightarrow y = x - a \), \( b = -a \), \( t \rightarrow \tau = \int_{t}^{T} \nu(s) ds \), \( G(t, x) \rightarrow F(\tau, y) = e^{-a} G(t, x) \) to obtain:

\[
\begin{align*}
-F_\tau + \frac{1}{2} F_{yy} + \mu F_y + \lambda \int_{-\infty}^{\infty} [F(y + J) - F] \varphi(J) dJ &= 0, \\
F(\tau, b) &= 0, F(0, y) = \max\{e^y - 1, 0\}.
\end{align*}
\]

The equation of this type was solved by Sepp (2004) through an application of Laplace transform with respect to \( \tau \), \( U(p, y) = \mathcal{L}[F(\tau, y)](p) \). We use his results (Proposition 5.1) to present the solution for \( U(p, y) \) as follows:

\[
U(p, y) = \begin{cases} 
C_0 e^{\psi_0 y} + C_1 e^{\psi_1 y} + A_2 e^{\psi_2 y} + A_3 e^{\psi_3 y}, y \leq 0 \\
C_2 e^{\psi_2 y} + C_3 e^{\psi_3 y} + A_3 e^{\psi_3 y} + \left[\frac{e^y}{p} - \frac{1}{p}\right], y > 0,
\end{cases}
\]

where constants \( C_0, C_1, C_2, C_3 \) are solution of the system

\[
\begin{pmatrix}
1 & 1 & -1 & -1 \\
\psi_0 & \psi_1 & -\psi_2 & -\psi_3 \\
\frac{1}{\psi_0 \eta + 1} & \frac{1}{\eta + 1} & -\frac{1}{\psi_2 \eta + 1} & -\frac{1}{\psi_3 \eta + 1} \\
\frac{1}{\psi_0 \eta + 1} & \frac{1}{\eta + 1} & -\frac{1}{\psi_2 \eta + 1} & -\frac{1}{\psi_3 \eta + 1}
\end{pmatrix}
\begin{pmatrix}
C_0 \\
C_1 \\
C_2 \\
C_3
\end{pmatrix}
= \begin{pmatrix}
0 \\
\frac{1}{p} \\
\frac{p(\eta + 1)}{p(\eta + 1)} - \frac{1}{p} \\
\frac{p(\eta + 1)}{p(\eta + 1)} + \frac{1}{p}
\end{pmatrix},
\]

and \( \psi_3, \psi_2, \psi_1, \psi_0 \) are solutions of the following quartic equation:

\[
\frac{1}{2} \eta^{-1} \psi^4 + \left(\mu \eta^{-1} - \frac{1}{2} (\eta^2 - \eta^4)\right) \psi^3 - \left(\frac{1}{2} + \mu (\eta^2 - \eta^4) + (p + \lambda) \eta^{-1} \eta^{-1}\right) \psi^2 \\
+ (-\mu + (p + \lambda) (\eta^2 - \eta^4) - \lambda (a^2 - a^4)) \psi + p = 0.
\]

which has four real roots \( \psi_i, i = 0, 1, 2, 3 \), such that

\[
-\infty < \psi_3 < -\frac{1}{\eta} < \psi_2 < 0 < \psi_1 < \frac{1}{\eta} < \psi_0 < \infty.
\]
The constants $A^\pm_{2,3}$ are computed by

$$A^\pm_{2,3} = \pm \frac{1 + \psi_2 \eta^-}{\psi_3 - \psi_2} \left[ (\psi_0 - \psi_3,2) e^{(\psi_0 - \psi_2,3)b} + (\psi_1 - \psi_3,2) e^{(\psi_1 - \psi_2,3)b} \right]. \quad (3.15)$$

As a result, the option value in the time domain is computed by means of the numerical inversion of the Laplace transform:

$$W(t, S) = (D(T) + K) e^{-\int_T^t r(s) ds} \mathcal{L}^{-1}[U(p, y)]. \quad (3.16)$$

To invert the Laplace transform, we can employ the algorithm by Stehfest (1970). If $U(p, y)$ is the Laplace transform of $V(\tau, y)$, then the original function can approximately be computed by

$$V(\tau, y) \approx \frac{\ln 2}{\tau} \sum_{j=1}^{\min\{j, N/2\}} \Lambda_j U\left(j \frac{\ln 2}{\tau}, y\right), \quad (3.17)$$

where coefficients $\Lambda_j$ are given by

$$\Lambda_j = (-1)^{N/2+j} \sum_{k=(j+1)/2}^{\min\{j, N/2\}} \frac{k^{N/2}(2k)!}{(N/2-k)!k!(k-1)!(j-k)!(2k-j)!}, \quad (3.18)$$

and $N$ is an even number and $k$ is computed using integer arithmetic.

VBA code for the numerical inversion of the Laplace transform by means of Stehfest algorithm is given in Sepp-Skachkov (2003). A nice property of the Stehfest algorithm is that since constants $\Lambda_j$ depend neither on $\tau$ nor on $y$ they can be tabulated and subsequently used for inversion of $U(\tau, y)$. In Table 1, we report values of $\Lambda_j$ for $N = 14$ using 8-digit accuracy (this choice allows to invert the original with accuracy up to four digits).

<table>
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<th>$j$</th>
<th>$\Lambda_j$</th>
<th>$j$</th>
<th>$\Lambda_j$</th>
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<td>14</td>
<td>-3925554.96666660</td>
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</table>

Table 1: Coefficients $\Lambda_j$ of the Stehfest algorithm for $N = 14$.

Computational speed of calculating call option prices for the double-exponential jump-diffusion is closely comparable to that for the diffusion with stochastic variance.
4 Survival Probability

In this section, we derive solutions for survival probability $Q(t, S)$ of the reference firm under the described models:

$$Q(t, S) = \mathbb{P}[S > T | \mathcal{F}_t] = \mathbb{E}^Q[1_{S(\tau) > 0: t < \tau \leq T} | \mathcal{F}_t]$$

which are necessary to calculate the values of credit default swaps.

4.1 Regular Diffusion

Under dynamics (2.6), $Q(t, S)$ satisfies the following PDE:

$$\begin{cases}
Q_t + \frac{1}{2} \nu(t)(S + D(t))^2 Q_{SS} + (r(t) - d(t)) S Q_S = 0, \\
Q(t, 0) = 0, \ Q(T, S) = 1.
\end{cases}$$

We introduce $y = \ln\left(\frac{S+D(t)}{D(t)}\right)$, $t \to \tau = \int_t^T \nu(s)ds$, $Q(t, S) \to F(\tau, y)$:

$$\begin{cases}
-F_\tau + \frac{1}{2} F_{yy} - \frac{1}{2} F_y = 0, \\
F(\tau, 0) = 0, \ F(0, y) = 1.
\end{cases}$$

We solve this equation by the method of images described in Appendix A1. The solution is given by:

$$F(\tau, y) = \mathcal{N}\left(\frac{y}{\sqrt{\tau}} - \frac{1}{2}\sqrt{\tau}\right) - e^y \mathcal{N}\left(-\frac{y}{\sqrt{\tau}} - \frac{1}{2}\sqrt{\tau}\right).$$

where $\mathcal{N}$ is the CDF of standard normal distribution.

For original variables, we have:

$$Q(t, S) = \mathcal{N}\left(\frac{\ln\left(\frac{S+D(t)}{D(t)}\right)}{\sqrt{\int_t^T \nu(s)ds}} - \frac{1}{2}\sqrt{\int_t^T \nu(s)ds} - \frac{S+D(t)}{D(t)} \mathcal{N}\left(\frac{\ln\left(\frac{D(t)}{S+D(t)}\right)}{\sqrt{\int_t^T \nu(s)ds}} - \frac{1}{2}\sqrt{\int_t^T \nu(s)ds}\right)\right).$$

The formula (4.5) is one of the key results in the CreditGrades model. We see that in this setup the survival probability has a few nice properties: 1) it does not depend on the risk-neutral drift, 2) it depends on the level of leverage $\frac{V(t)}{D(t)}$ rather than on $V(t)$ and $D(t)$ directly.

4.2 Diffusion with Stochastic Volatility

Under this model, $Q(t, S)$ solves the following equation:

$$\begin{cases}
Q_t + \frac{1}{2} \nu(t)(S + D(t))^2 Q_{SS} + (r(t) - d(t)) S Q_S + \kappa \nu(\nu_\infty - \nu) Q_\nu + \frac{1}{2} \varepsilon \nu Q_{\nu\nu} = 0, \\
Q(t, 0) = 0, \ Q(T, S) = 1.
\end{cases}$$

12
Introducing $y = \ln\left(\frac{S+D(t)}{D(t)}\right)$, $t \rightarrow \tau = T - t$, $Q(t, S) \rightarrow Z(\tau, y) = e^{-y/2}Q(t, S)$, we obtain:

$$
\begin{align*}
-\tau Z + \frac{1}{2} \nu Z_{yy} - \frac{1}{2} \nu Z_y + \kappa \nu (\nu_\infty - \nu) Z_\nu + \frac{1}{2} \varepsilon \nu Z_{\nu \nu} - \frac{1}{8} \nu Z &= 0, \\
Z(\tau, 0) &= 0, \quad Z(0, y) = e^{-\frac{y}{2}}.
\end{align*}
\tag{4.7}
$$

We solve this PDE via a combination of the Fourier transform and the method of images described in Appendix A2. For this case, we compute the transform of the initial data by:

$$
\Xi(y, k) = \int_0^\infty e^{-\frac{1}{2} y} \Upsilon(\tau, y', y, \nu) dy' - \frac{\nu A(\nu(\tau, k) + B(\nu(\tau, k)))}{ik - \frac{1}{2}} + \frac{\nu B(\nu(\tau, k) + B(\nu(\tau, k)))}{ik - \frac{1}{2}} = I(y, k),
\tag{4.8}
$$

where $\Upsilon(\tau, y', y, \nu)$ is defined by (10.5), and $A(\nu(\tau, k))$, $B(\nu(\tau, k))$ are defined by (10.4).

Finally, $Z(\tau, y)$ is computing by inverting the transform:

$$
Z(\tau, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Xi(y, k) dk = \frac{1}{\pi} \int_0^{\infty} \Re[I(y, k)] dk = \frac{2}{\pi} \int_0^{\infty} \frac{e^{\nu(\nu(\tau, k) + B(\nu(\tau, k)))\nu} k \sin(\nu k)}{k^2 + \frac{1}{4}} dk.
\tag{4.9}
$$

Given the value of $Z(\tau, y)$, we compute $Q(t, S)$ by:

$$
Q(t, S) = e^{\frac{1}{2} y} Z(\tau, y).
\tag{4.10}
$$

Thus, the problem boils down to computing numerically integral in (4.9), which is straightforward to implement.

### 4.3 Double-Exponential Jump-Diffusion

Under this process, $Q(t, S)$ solves the following PIDE:

$$
\begin{align*}
Q_t + \frac{1}{2} \nu(t)(S + D(t))^2Q_{ss} + (r(t) - d(t)) SQ_s + \\
+ \lambda \nu(t) \int_{-\infty}^{\infty} [Q((S + D(t))e^J) - Q(S + D(t))] \varpi(J) dJ &= 0, \\
Q(t, 0) = 0, \quad Q(T, S) = 1.
\end{align*}
\tag{4.11}
$$

Similarly, we introduce $y = \ln\left(\frac{S+D(t)}{D(t)}\right)$, $t \rightarrow \tau = \int_t^T \nu(s) ds$, $Q(t, S) \rightarrow F(\tau, y)$ to obtain:

$$
\begin{align*}
-F_\tau + \frac{1}{2} F_{yy} + \mu F_y + \lambda \int_{-\infty}^{\infty} [F(x + J) - F] \varpi(J) dJ &= 0, \\
F(\tau, 0) = 0, \quad F(0, y) = 1.
\end{align*}
\tag{4.12}
$$

with $\mu = -\frac{1}{2} - \lambda \alpha$.

The solution to the Laplace transform of this equation $U(p, y) = \mathcal{L}[F(\tau, y)](p)$ is given by:

$$
U(p, y) = A_2 e^{\nu_2 y} + A_3 e^{\nu_3 y} + \frac{1}{p},
\tag{4.13}
$$
where
\[ A_2 = \frac{-1}{p} \psi_3 (1 + \eta - \psi_2), \quad A_3 = \frac{1}{p} \psi_2 (1 + \eta - \psi_3), \]
and \( \psi_2, \psi_3 \) are corresponding roots of polynomial (3.14).

In the time domain, the survival probability is computed via the inversion of the Laplace transform:
\[ Q(t, S) = \mathcal{L}^{-1}[U(p, y)], \]
where we employ Stehfest algorithm to invert \( U(p, y) \).

### 4.4 Default (First-Exit) Time Density

Having derived the closed-form solutions for the survival probability, we are now able to calculate the density of the firm’s default time, which is useful to take into account. By definition, this density, \( q(t, S) \), satisfies:
\[ q(t, S)dt = \mathbb{P}^Q[\xi \in (T, T + dt)|\mathcal{F}_t] = \frac{d}{dT} (1 - Q(t, S)) = -\frac{d}{dT} Q(t, S). \]

For regular diffusion we use formula (4.5) to obtain:
\[ q(t, S) = \nu(T) \ln \left( \frac{S + D(t)}{D(t)} \right) \left( \frac{\ln \left( \frac{S + D(t)}{D(t)} \right)}{\sqrt{\int_t^T \nu(s)ds}} - \frac{1}{2} \sqrt{\int_t^T \nu(s)ds} \right), \]
where \( n() \) is the PDF of standard normal random variable.

Similarly, for diffusion with stochastic volatility we use formula (4.10) (the differentiation inside the integral sign is allowed since the integral converges uniformly) to obtain:
\[ q(t, S) = e^{\frac{1}{2}y^2} \frac{2}{\pi} \int_0^\infty \left( A'(T) + B'(T)\nu \right) e^{\frac{A(T-t,k) + B(T-t,k)\nu_k}{k^2 + \frac{1}{4}}} \sin(ky) dk, \]
with
\[ B'(T) = -(k^2 + \frac{1}{4}) \frac{2\zeta^2 e^{-(T-t)\zeta}}{(\psi^- + \psi^+ e^{-(T-t)\zeta})^2}, \]
\[ A'(T) = -\frac{\kappa \nu \infty}{\varepsilon^2} \left[ \frac{\psi^+}{\psi^-} + 2 \frac{\psi^+ e^{-(T-t)\zeta}}{\psi^- + \psi^+ e^{-(T-t)\zeta}} \right]. \]

Finally, for double-exponential jump-diffusion we employ the following relationship:
\[ \mathcal{L}[q(t, S)] = -p\mathcal{L}[Q(t, S)], \]
and use formula (4.13) to compute \( q(t, S) \) by inverting the Laplace transform:
\[ q(t, S) = -\nu(T)\mathcal{L}^{-1}[pU(p, y)]. \]
In Figure (4.4), we show the default (first-exit) probabilities for the respective diffusions with parameters obtained by calibrating these models to General Motors data (to be discussed in Section 7, parameter values are given in Table 2). We see that eventually the firms default with probability one (let us recall that the Brownian motion hits every level with probability one). Since the exit densities have heavy right tails, the actual time of default can take place in a very distant future (let us recall that the first-exit time for Brownian motion is a finite random variable but it has an infinite mean with probability one - see, for example, Karatzas-Shreve (1991), Remark 8.3).

![Figure 4.1: The default (first-exit) probabilities are shown on the left side. The corresponding default (exit-time) densities are shown on the right side. Model parameters are taken from Table 2 for General Motors data. DEJD stands for double-exponential jump-diffusion, RD stands for regular diffusion, and SV stands for diffusion with stochastic volatility. The exit densities have heavy right tails.](image)

Let us also note that the mode of the double-exponential jump-diffusion is less than the modes of regular and stochastic volatility diffusions. Thus, the jump-diffusion implies higher default probabilities for short-term maturities, which is in agreement with empirical observations.

5 Random Default Barriers

In this section we consider random default barriers. In the CreditGrades model, random barriers are intended to introduce uncertainty to the default event. In this formulation the default barrier is given by $D(t) = RB(t)$, where $R$ is a log-normal random variable independent of the dynamics of $V(t)$ with the expected value $\overline{R}$, drift $-\frac{1}{2}\beta^2$ and standard derivation $\beta$. The formula for the firm’s survival probability is based on an approximation of a Brownian motion with a random barrier by another Brownian motion with a flat barrier and an appropriately chosen diffusion parameter and initial value.

From (2.10), we see that default does not occur as long as for all $0 \leq t \leq T$

$$V(0)e^{-\frac{1}{2}\int_0^t \nu(s)ds + \int_0^t \sqrt{\nu(s)}dZ(s)} > \overline{RB}(0)e^{-\frac{1}{2}\beta^2 + \beta Y},$$

(5.1)
where $Y$ is a standard normal random variable independent of the Brownian motion $Z(t)$. In this setting, $Y$ is interpreted as a time-independent state variable whose value is revealed at time $t = 0$. By introducing a process

$$X(t) = \int_0^t \sqrt{\nu(s)}dZ(s) - \beta Y - \frac{1}{2} \int_0^t \nu(s)ds - \frac{\beta^2}{2},$$  \hspace{1cm} (5.2)

we can present (5.1) as:

$$X(t) > \ln \frac{RB(0)}{V(0)} - \beta^2 = - \ln \frac{V(0)e^{\beta^2}}{RB(0)}.$$  \hspace{1cm} (5.3)

It follows that $X(t)$ is normally distributed with

$$\mathbb{E}[X(t)] = - \frac{1}{2} \int_0^t \nu(s)ds - \frac{1}{2} \beta^2 = - \frac{\int_0^t \nu(s)ds}{2t} \left( t + \frac{\beta^2 t}{\int_0^t \nu(s)ds} \right),$$

$$\mathbb{V}[X(t)] = \int_0^t \nu(s)ds + \beta^2 = \frac{\int_0^t \nu(s)ds}{t} \left( t + \frac{\beta^2 t}{\int_0^t \nu(s)ds} \right).$$  \hspace{1cm} (5.4)

Then process $X(t)$ is approximated with a Brownian motion $\hat{X}(t)$ with drift $-\frac{\int_0^t \nu(s)ds}{2t}$ and variance $\frac{\int_0^t \nu(s)ds}{t}$ starting in the past at $-\Delta t = -\frac{\beta^2 t}{\int_0^t \nu(s)ds}$ with $\hat{X}(-\Delta t) = 0$. Under these assumptions, to calculate the survival probability we can use formula (4.4) with $\tau = \int_t^T \nu(s)ds + \beta^2$ and $y = \frac{V(t)e^{\beta^2}}{RB(t)}$.

Finger et al (2002, p.8) mention that "this approximation replaces the uncertainty in the default barrier with an uncertainty in the level of the asset value at time t = 0". These approximation has its own limitations as pointed out by Finger et al: it includes the possibility of default in the period $(-\Delta t, 0]$, which leads to unrealistic result that there is a non-zero probability of default at $t = 0$. In addition, it is unclear how to implement this procedure for pricing equity options and for using diffusions with stochastic variance and jumps.

A viable alternative is to use the assumption that $R$ is independent of the firm’s value dynamics and employ the conditioning on $R$ to represent the survival probability with a random barrier, $\Theta(t, V)$, by:

$$\Theta(t, V) = \mathbb{E}^Q[\mathbf{1}_{\{V(T) > RB(T)\}}]\mathbb{Q} = \int_0^\infty \mathbb{E}^Q[\mathbf{1}_{\{V(t) > R',RB(t)\}}]\mathbb{Q}\mu_R(R')dR',$$  \hspace{1cm} (5.5)

where $\mu_R$ is PDF of $R$ and $Q(t, V, R) = Q(t, S)$ is the survival probability assuming a fixed value of $R$.

Next, we expand $Q(t, V, R)$ in Taylor series around the expected value of $R$, $\overline{R}$:

$$Q(t, V, R) = Q(t, V, \overline{R}) + Q_R(t, V, \overline{R})(R - \overline{R}) + \frac{1}{2} Q_{RR}(t, V, \overline{R})(R - \overline{R})^2$$

$$+ \frac{1}{6} Q_{RRR}(t, V, \overline{R})(R - \overline{R})^3 + \frac{1}{24} Q_{RRRR}(t, V, \overline{R})(R - \overline{R})^4 + ...$$  \hspace{1cm} (5.6)
Accordingly, we can use the convexity adjustment and approximate $\Theta(t, V)$ by

$$\Theta(t, V) \approx Q(t, V, \mathcal{R}) + \frac{1}{2}Q_{RR}(t, V, \mathcal{R})\vartheta_2 + \frac{1}{6}Q_{RRR}(t, V, \mathcal{R})\vartheta_3 + \frac{1}{24}Q_{RRRR}(t, V, \mathcal{R})\vartheta_4,$$

(5.7)

where $\vartheta_2$, $\vartheta_3$, $\vartheta_4$ are the corresponding central moments of $R$ given by:

$$\vartheta_2 = \mathcal{R}^2(e^{\beta^2} - 1), \quad \vartheta_3 = \mathcal{R}^3(e^{\beta^2} - 1)^2(e^{\beta^2} + 2), \quad \vartheta_4 = \mathcal{R}^4(e^{\beta^2} - 1)^2(e^{4\beta^2} + 2e^{3\beta^2} + 3e^{2\beta^2} - 3).$$

(5.8)

Partial derivatives in (5.7) can be approximated numerically with high accuracy given the analytical solutions for $Q(t, V, R)$. In general, the approximation of $N$th order derivative by a differencing formula with second order accuracy requires $N + 1$ evaluations of the given function. Thus, by using formula (5.7), we need 5 evaluations of $Q(t, V, R)$, which is not burdensome since we use an explicit expression for $Q(t, V, R)$.

In Figure (5.1) we illustrate incorporating random default barrier using CreditGrades approximation (CG), convexity formula (5.7) with 2 terms (2t) and with 4 terms (4t) for calculating CDS spreads using formula (6.4) under the regular diffusion. The spread implied by the diffusion with constant barrier $\mathcal{R}$ (RD) is given for comparison purposes. The term structure of variance is given by formula (7.1) and all relevant parameters are taken from Table 2 in Section 7 to make our analysis consistent with General Motors data.

![Figure 5.1: CDS spreads for $\beta = 0.2$ are shown on the left side. The CDS spreads for $\beta = 0.12$ are shown on the right side. Other parameters are taken from Table 2 for General Motors data: $\mathcal{R} = 0.5$, $S(0) = 25.86$, $B(0) = 65$, $V(0) = S(0) + \mathcal{R}B(0) = 58.36$, $\nu_0 = 0.11$, $\nu_\infty = 0.05$, $\kappa_\nu = 3.57$. In CreditGrades approximation, higher values of $\beta$ imply non-zero default probability at time $t = 0$ leading to unrealistic CDS spreads for very short maturities. For moderate values of $\beta$, all three approximations are comparable. It follows that for larger values of $\beta$, the CreditGrades approximation can lead to nonzero default probability at time $t = 0$ and, as a result, to unrealistically high shorter-term spreads. For moderate values of $\beta$, CDS spreads are basically equivalent, especially, for longer maturities. In addition, we see that random default spread is higher in the CreditGrades approximation than in the other two approximations.](image-url)
barriers lead to higher shorter-term spreads compared to fixed barriers. Thus, random barriers naturally introduce the unpredictability of default and result in higher short-term spreads. Furthermore, mixing random barriers with stochastic variance or jumps can lead to a better modeling of shorter-term spreads.

The advantage of using the convexity adjustment formula (5.7) is that it can be extended in a straightforward way for calculating survival probabilities as well as option prices under the diffusion with stochastic variance and double-exponential jump-diffusion.

Let us mention a potential drawback of using log-normally distributed default barrier. Since lognormally distributed random variable can take any positive values, there might be a positive and significant probability that $R > 1$ or, equivalently, that the default barrier is greater than the firm’s debt, which is quite unrealistic and counterintuitive consequence. To circumvent this problem, we can assume that $R$ has beta distribution $\text{B}(a,b)$ with parameters $a$ and $b$ chosen to ensure that $\mathbb{E}[R] = \bar{R}$ and $\mathbb{V}[\ln(R)] = \beta^2$. The beta distribution is easy to handle by using the convexity adjustment formula (5.7).

6 Pricing Credit and Equity Default Swaps

In this section we consider pricing of two very popular credit derivative claims: credit and equity default swaps. We note that one of the key purposes of developing CreditGrades model was to replicate the CDS spreads observed in the market.

6.1 Credit Default Swaps

We consider a continuous-time CDS contract, in which the holder is long the default protection of maturity $T$ and pays, conditional on the survival of the reference company, to the writer fixed payment $\Delta_{\text{CDS}}(T)\mathcal{N}_C 1_{\{\varsigma > T\}}$, where $\Delta_{\text{CDS}}(T)$ is par credit spread and $\mathcal{N}_C$ is notional.

We can present the fixed leg of the default swap as

$$\text{Fixed Leg} = \mathbb{E}^Q \left[ \int_t^T Y(t,u)\Delta_{\text{CDS}}(T)\mathcal{N}_C 1_{\{\varsigma > u\}}du \right] = \Delta_{\text{CDS}}(T)\mathcal{N}_C \int_t^T Y(t,u)Q(u,S)du, \quad (6.1)$$

where $Y(t,T) = e^{-\int_t^T r(s)ds}$ is the risk-free discount bond.

The writer of the CDS will pay recovery value $\mathcal{N}_C \left(1 - R_C \right) 1_{\{t < \varsigma \leq T\}}$ at default time $\varsigma$. Thus, the floating leg of CDS can be presented as:

$$\text{Floating Leg} = \mathbb{E}^Q \left[ \int_t^T Y(t,u)\left(1 - R_C \right)\mathcal{N}_C 1_{\{\varsigma < u\}}du \right] = -\left(1 - R_C \right)\mathcal{N}_C \int_t^T Y(t,u)dQ(u,S) \quad (6.2)$$

$$= \left(1 - R_C \right)\mathcal{N}_C \left(1 - Y(t,T)Q(T,S) + \int_t^T Q(u,S)dY(t,u) \right).$$
Par spread spread $\Delta_{CDS}(T)$ is chosen to equate both legs, thus $\Delta_{CDS}(T)$ can be calculated by:

$$\Delta_{CDS}(T) = (1 - R_C) \left( 1 - Y(t, T)Q(T, S) + \int_t^T Q(u, S)dY(t, u) \right) / \int_t^T Y(t, u)Q(u, S)du. \quad (6.3)$$

If the interest rate is constant ($Y(t, T) = e^{-(T-t)r}$), then the above formula reduces to:

$$\Delta_{CDS}(T) = (1 - R_A) \left( 1 - e^{-(T-t)r}Q(T, S) \int_t^T e^{-(u-t)r}Q(u, S)du - r \right). \quad (6.4)$$

### 6.2 Equity Default Swaps

In an equity default swaps (EDS) the buyer of the protection with maturity $T$ pays a fixed payment $\Delta_{EDS}(T)N_E$ provided the spot price does not hit a prescribe barrier level during contract inception and maturity dates $((t, T])$. Now let $\Delta_{EDS}(T)$ denote EDS spread and $N_E$ denote its notional. The barrier level is set at $D_{EDS} = cS(0)$, where usually $c = 0.7$ or $c = 0.5$. If the spot price hits the barrier, the writer of the EDS will pay recovery value $R_E N_E$, $0 \leq R_E \leq 1$.

Let, $Q_c(t, S)$ denote the probability that the spot price process will not reach the barrier $cS(0)$ up to maturity time $T$:

$$Q_c(t, S) = \mathbb{P}_t[S(\tau) > cS(0), \tau \in (t, T)|F_t]. \quad (6.5)$$

EDS par spread $\Delta_{EDS}(T)$ can be express in terms of $Q_c(t, S)$ by analogy with formulas (6.3) or (6.4). Thus the problem of calculating $\Delta_{EDS}(T)$ boils down to finding an explicit expression for $Q_c(t, S)$. Now, the problem becomes a little complicated since we are dealing with a flat barrier. For simplicity, we assume that $r(t) = d(t)$, then we will be able to use survival probabilities obtained in Section 4 where now we replace $y$ by $y_c = \ln \left( \frac{S + D(t)}{cS(0) + D(t)} \right)$. In case $r(t) \neq d(t)$, we can still obtain close-form solution to $Q_c(t, S)$ assuming constant risk-free and dividend rates or if the triggering barrier is time-dependent with the following structure: $D_{EDS}(t) = cS(0)e^{\int_t^T (r(s) - d(s))ds}$.

### 7 Calibration Results

By applying structural models, one needs to obtain accurate estimates for the current firm’s asset value $V(0)$ and firm’s volatility. Hull at al (2005) and Stamicar-Finger (2005) use equity option data to back out these estimates. Specifically, Hull at al (2005) use 50- and 25-delta implied put volatilities of options with maturity two month, while Stamicar-Finger (2005) suggest three methods to obtain these estimates: (a) estimate firm’s debt per share $B(0)$ from balance sheet data and imply asset volatility from a one-year at-the-money (ATM) volatility, (b) use a one-year ATM volatility and a CDS spread quote to back out both $B(0)$ and asset
volatility, (c) use two one-year implied volatilities corresponding to 50 and 75 deltas to imply $B(0)$ and asset volatility.

The purpose of our calibration is to show the ability of presented models to fit the implied volatility surface and compare their produced volatility surfaces. Since parameters for diffusions with stochastic volatility and double-exponential jumps are not directly observable, we have to use a calibration procedure to back out these parameters from options data. Although this procedure is more involved, it has an advantage to fit a model to the whole implied volatility surface in addition to some available CDS quotes, and thus it enables us to use all available data to estimate and manage default risk.

To illustrate our models, we calibrate them to General Motors option data. By calibration, we follow procedure (a) suggested by Stamicar-Finger (2005): we first estimate firm’s debt per share $B(0)$ from balance sheet data and then back out model parameters from options implied volatilities. Option implied volatilities were collected from Bloomberg on November 8, 2005. The spot price is $S(0) = 25.86$. Dividend yield reported by Bloomberg is $d = 0.078$. Other data are given in Table (3) of Appendix A.3.

An estimate for ratio of total debt per equity, 12.4, is taken from General Motors balance sheet. This gives that an approximate debt per share is $320$. However, as pointed out by Stamicar-Finger (2005), over 80% of outstanding GM debt is issued by its financial services subsidiary (General Motors Acceptance Corporation (GMAC)), and since GMAC operates like a financial institution, much of its debt is secured. They also report that since July 2003 the implied debt-per-share for GM has been approximately 20%-25% of its total debt-per-share which includes all liabilities of GM and its subsidiaries. Following these remarks, we take $65$ as an estimate of GM debt-per-share and assume that $R = 0.5$. Accordingly, debt-per-share is taken to be: $D(0) = 32.5$.

For regular diffusion and double-exponential jump-diffusion, we assume that the firm’s asset variance is time-inhomogeneous:

$$\nu(t) = \nu_{\infty} + (\nu_0 - \nu_{\infty})e^{-\kappa_{\nu}t}, \quad (7.1)$$

where $\nu_0$ is an initial variance, $\nu_{\infty}$ is a long-term mean, and $\kappa_{\nu}$ is a reversion speed to the long-term mean.

Parameter estimates are reported in Table 2. DEJD stands for double exponential-jump-diffusion, RD stands for regular diffusion, SV stands for diffusion with stochastic variance. In Figures (7.1), (7.2), and (7.3), we show produced implied volatility surfaces and differences between model and market volatilities. Since for shorter maturities we do not have volatility quotes for all the strikes, these differences are computed only for available market volatilities, while the model implied volatilities are computed for all given strikes and maturities.

It follows that the regular diffusion and diffusion with stochastic variance imply approximately the same estimates for initial variance, $\nu_0$, and long-term variance, $\nu_{\infty}$, while higher value for mean-reversion speed, $\kappa_{\nu}$, in stochastic variance model accounts for a high volatility of variance, $\varepsilon_{\nu}$. Lower values for $\nu_0$ and $\nu_{\infty}$ in double-
Table 2: Parameter estimates backed out from GM options data

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<th>SV</th>
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<td>$\nu_0$</td>
<td>0.0260 (0.1369$^2$)</td>
<td>0.1051 (0.3242$^2$)</td>
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</tr>
<tr>
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<tr>
<td>$\varepsilon_\nu$</td>
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</table>

exponential model can be explained that some uncertainty is introduced by jump risk, whose initial intensity is $\lambda \nu_0 = 4.2308$ and long-term intensity is $\lambda \nu_\infty = 2.4539$.

From Figures (7.1), (7.2), and (7.3), we see that both double-exponential jump-diffusion and diffusion with stochastic variance provide a good fit to the data and yield steep volatility surfaces.

In Figure (7.4), we show the survival probabilities and corresponding CDS spreads implied by the models. We see that both double-exponential jump-diffusion and diffusion with stochastic variance imply high short-term spreads, which is in good agreement with empirical observation. The double-exponential jump-diffusion produces higher short-term spreads compared to those implied by the diffusion with stochastic variance, which can be explained that even if the firm’s variance is uncertain the diffusion needs time to reach the default barrier. The implied time of reaching the default barrier for the diffusion with stochastic variance is less than that for the regular diffusion, which makes the short-term spreads higher.

In Figure (7.5), we show probabilities of hitting the barrier set at $cS(0)$ with $c = 0.5$ and corresponding EDS spreads implied by the models.
Figure 7.1: The implied volatility surface of regular diffusion is shown on the left side. Differences between market and model implied volatility are shown on the right side. The regular diffusion introduces the volatility skew, but it is unable to produce the volatility smile. Also, it fits the ATM volatility structure relatively well.

Figure 7.2: The diffusion with stochastic variance introduces both the volatility skew and smile. The differences are relatively big for lower and higher strikes.

Figure 7.3: The double-exponential jump diffusion is capable of introducing both the volatility smile and volatility skew. It provides a good fit to the whole implied volatility surface.
Figure 7.4: The survival probabilities are shown on the left side. The corresponding CDS spreads are shown on the right side. Parameters are taken from Table 2 assuming $R_C = 0$. Double-exponential diffusion and diffusion with stochastic variance imply higher short-term spreads consistent with market observations.

Figure 7.5: Probabilities of hitting the barrier level $cS(0)$ with $c = 0.5$ are shown on the left side. The corresponding EDS spreads are shown on the right side. Parameters are taken from Table 2, assuming $r = d = 0.078$ and $R_E = 0$.

8 Conclusions

We considered two alternative models to the pure diffusion model underlying the CreditGrades approach to the default risk. One model is based on the diffusion with stochastic variance and another one is based on the double-exponential jump-diffusion. We obtained close-form solutions for call option values and survival probabilities under these models and applied them to modeling CDS and EDS spreads. Also, we applied a convexity adjustment formula to incorporate random default barriers into the proposed models. We calibrated these models to General Motors option data and showed that, firstly, these models are in a good agreement with implied volatility surface and, secondly, they produce high shorter-term spreads consistent with market observations.
9 Appendix A.1. Derivation of formula (3.3) for the value of a call option under the regular diffusion (2.6).

We solve PDE (3.2) in a few steps.

1) We introduce new variables as follows:

\[
x = \ln \frac{S + D(t)}{D(t)}, \quad a = \ln \frac{D(T) + K}{D(T)}, \quad W(t, S) \to G(t, x) = e^{\int_0^T r(s)ds} \frac{W(t, S)}{D(T)}.
\]

Then \(G(t, x)\) solves the following equation:

\[
\begin{cases}
G_t + \frac{1}{2} \nu(t) G_{xx} - \frac{1}{2} \nu(t) G_x = 0, \\
G(t, 0) = 0, \quad G(T, x) = \max\{ e^x - e^a, 0 \}.
\end{cases}
\]

(9.2)

2) We introduce new variables as follows:

\[
x \to y = x - a, \quad b = -a, \quad t \to \tau = \int_t^T \nu(s)ds, \quad G(t, x) \to F(\tau, y) = e^{-a} G(t, x).
\]

(9.3)

Now, \(F(\tau, y)\) solves:

\[
\begin{cases}
-\tau F_\tau + \frac{1}{2} F_{yy} - \frac{1}{2} F_y = 0, \\
F(0, y) = 0, \quad F(\tau, 0) = \max\{ e^y - 1, 0 \}.
\end{cases}
\]

(9.4)

3) Finally, we introduce \(F(t, x) \to Z(\tau, y) = e^{\tau/8 - y/2} F(\tau, y)\):

\[
\begin{cases}
-\tau Z_\tau + \frac{1}{2} Z_{yy} = 0, \\
Z(\tau, b) = 0, \quad Z(0, y) = \max\{ e^{\frac{y}{2}} - e^{-\frac{y}{2}}, 0 \}.
\end{cases}
\]

(9.5)

4) We solve this equation by the method of images:

\[
\Upsilon(\tau, y, y') = H(\tau, y - y') - H(\tau, y - (2b - y')),
\]

(9.6)

where \(H(\tau, Y)\) is the heat kernel:

\[
H(\tau, Y) = \frac{e^{-Y^2/4\tau}}{\sqrt{2\pi\tau}}.
\]

(9.7)

Then \(Z(\tau, y)\) is computed by:

\[
Z(\tau, y) = \int_0^\infty (e^{\frac{y'}{2\tau}} - e^{-\frac{y'}{2\tau}}) \Upsilon(\tau, y, y') dy' =
\]

\[
e^{\tau/8 + y/2} \mathcal{N}\left(\frac{y}{\sqrt{\tau}} + \frac{1}{2\sqrt{\tau}}\right) - e^{\tau/8 - y/2} \mathcal{N}\left(\frac{y}{\sqrt{\tau}} - \frac{1}{2\sqrt{\tau}}\right) -
\]

\[
e^{\tau/8 + b - y/2} \mathcal{N}\left(\frac{2b - y}{\sqrt{\tau}} + \frac{1}{2\sqrt{\tau}}\right) + e^{\tau/8 - b + y/2} \mathcal{N}\left(\frac{2b - y}{\sqrt{\tau}} - \frac{1}{2\sqrt{\tau}}\right).
\]

(9.8)
Substituting original variables and using the obvious relationship
\[ \ln \frac{D(T)}{D(t)} = \int_t^T (r(s) - d(s))\,ds, \]
we obtain:
\[
W(t, S) = e^{-\int_t^T r(s)\,ds} \frac{S(t) + D(t)}{D(t)} \mathcal{N}(q_1^+) - e^{-\int_t^T r(s)\,ds} (D(T) + K) \mathcal{N}(q_1^-)
\]
\[ - e^{-\int_t^T r(s)\,ds} D(t) \mathcal{N}(q_2^+) + e^{-\int_t^T r(s)\,ds} \frac{(D(T) + K)(S(t) + D(T))}{D(t)} \mathcal{N}(q_2^-), \]
(9.9)

where
\[
q_1^{+-} = \frac{\ln \frac{S(t) + D(t)}{K + D(T)} + \int_t^T (r(s) - d(s))\,ds}{\sqrt{\int_t^T \nu(s)\,ds}} \pm \frac{1}{2} \sqrt{\int_t^T \nu(s)\,ds},
\]
\[
q_2^{+-} = \frac{\ln \frac{D^2(t)}{(K + D(T))(S(t) + D(T))} + \int_t^T (r(s) - d(s))\,ds}{\sqrt{\int_t^T \nu(s)\,ds}} \pm \frac{1}{2} \sqrt{\int_t^T \nu(s)\,ds}. \]
(9.10)

After a little rearrangement, we obtain:
\[
W(t, S) = e^{-\int_t^T r(s)\,ds} \frac{S(t) + D(t)}{D(t)} \mathcal{N}(q_1^+) - e^{-\int_t^T r(s)\,ds} \left( \frac{D^2(t)}{S(t) + D(T)} \mathcal{N}(q_2^+) - e^{-\int_t^T r(s)\,ds} (D(T) + K) \mathcal{N}(q_2^-) \right). \]
(9.11)

The first two terms represent the value of a call option with spot price \( S(t) + D(t) \) and strike \( D(T) + K \), while the last two terms represent the value of a call option with spot price \( \frac{D^2(t)}{S(t) + D(T)} \) and strike \( D(T) + K \). Accordingly, we obtain the formula (3.3).

### 10 Appendix A.2. Derivation of formula (3.5) for the value of a call option under the diffusion with stochastic variance (2.13).

We solve PDE (3.4) as follows.

1) We apply the sequence of transformations (9.1) and (9.3) with \( t \rightarrow \tau = T - t \) to obtain the following equation for \( F(\tau, y) \):
\[
\begin{cases}
-F_{\tau} + \frac{1}{2} \nu F_{yy} - \frac{1}{2} \nu F_y + \kappa_\nu (\nu_{\infty} - \nu) F_\nu + \frac{1}{2} \varepsilon_\nu \nu F_{\nu\nu} = 0, \\
F(\tau, 0) = 0, \quad F(0, y) = \max \{ e^y - 1, 0 \}. 
\end{cases}
\]
(10.1)

2) Next we introduce \( F(t, x) \rightarrow Z(\tau, y) = e^{-y/2} F(\tau, y) \):
\[
\begin{cases}
-Z_{\tau} + \frac{1}{2} \nu Z_{yy} - \frac{1}{2} \nu Z_y + \kappa_\nu (\nu_{\infty} - \nu) Z_\nu + \frac{1}{2} \varepsilon_\nu \nu Z_{\nu\nu} - \frac{1}{8} \nu Z = 0, \\
Z(\tau, 0) = 0, \quad Z(0, y) = \max \{ e^y - e^{-y/2}, 0 \}.
\end{cases}
\]
(10.2)
We solve PDE (10.2) via a combination of the Fourier transform and the method of images. A similar technique was employed by Lipton (2001) for pricing double-barrier options in the Heston’s model with zero correlation. It is well-known that the Fourier transform of Greens function for unbounded solution of PDE (10.2) has the affine form (for example, see Lewis (2000), Lipton (2001), Kangro at al. (2004)):

$$H(\tau,Y,\nu) = \int_{-\infty}^{\infty} e^{ikY+A(\tau,k)+B(\tau,k)\nu} dk,$$

with $Y = y' - y$, and

$$A(\tau,k) = -\frac{\kappa\nu}{\varepsilon^2} \left[ \psi_+ \tau + 2 \ln \left( \frac{\psi_+ + \psi_-}{2\zeta} \right) \right],$$

$$B(\tau,k) = -\left( \frac{k^2}{4} + 1 \right) \psi_+ + \psi_- e^{-\zeta\tau},$$

$$\psi_\pm = \pm \kappa \nu + \zeta, \zeta = \sqrt{\kappa^2 + \varepsilon^2 \nu \left( \frac{k^2}{4} + 1 \right)}.$$

We note that $H(\tau,y' - y,\nu)$ is invariant with respect to the reflections $y, k \rightarrow -y, -k$, so we can employ the method of images to find a bounded solution reflected at $x = b$:

$$\Upsilon(\tau,y',y,\nu) = H(\tau,y' - y,\nu) - H(\tau,y' + y - 2b,\nu).$$

Then we compute the transform of the initial data by:

$$\Xi(y,k) = \int_0^\infty \left( e^{\frac{1}{2}y'} - e^{-\frac{1}{2}y'} \right) \Upsilon(\tau,y',y,\nu) dy'$$

$$= e^{A(\tau,k)+B(\tau,k)\nu} \left( e^{-y'ik} - e^{(y-2b)ik} \right) \left( -\frac{1}{ik + \frac{1}{2}} + 2\pi \delta(k - \frac{1}{2}i) + \frac{1}{ik - \frac{1}{2}} \right)$$

$$= 2\pi \left( e^{\frac{1}{2}y} - e^{-\frac{1}{2}(y-2b)} \right) - e^{A(\tau,k)+B(\tau,k)\nu} \left( e^{-y'ik} - e^{(y-2b)ik} \right) \frac{k^2 + \frac{1}{4}}{k^2 + \frac{1}{4}}$$

$$= 2\pi \left( e^{\frac{1}{2}y} - e^{-\frac{1}{2}(y-2b)} \right) - I(\tau,k),$$

where we employed the celebrated formula for the complex exponential:

$$\int_0^\infty e^{(ik+\gamma)y} dy = -\frac{1}{ik + \gamma} + \left\{ \begin{array}{ll} 0, & \gamma < 0, \\
2\pi \delta(k - i\gamma), & \gamma > 0. \end{array} \right.$$  

Finally, $Z(\tau,y)$ can be computing by inverting the transform:

$$Z(\tau,y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Xi(y,k) dk = e^{\frac{1}{2}y} - e^{-\frac{1}{2}(y-2b)} - \frac{1}{\pi} \int_0^\infty \Re[I(y,k)] dk.$$

After a little simplification, we obtain a one dimension real-valued integral given in (3.6).
11 Appendix A.3. General Motors Option Data

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Table 3: Implied Volatilities of General Motor Options

References


